JOURNAL OF AIRCRAFT
Vol. 35, No. 2, March–April 1998

Study on Crowded Two-Dimensional Airspace—Self-Organized Criticality

Kuo-Chi Lin*
Institute for Simulation and Training, Orlando, Florida 32826-0544
Alex Sisti†
U.S. Air Force Research Laboratory, Rome, New York 13441-4505
and
Lee Chow‡
University of Central Florida, Orlando, Florida 32816-2385

A crowded airspace involves large numbers of airplanes interacting with each other. In general, the dynamic behavior of the system is quite complex and conventional mathematical modeling is difficult. Here, a generic air traffic model is used to investigate the application of the notion of self-organized criticality. An example of a crowded two-dimensional airspace near an airport is studied. The results show that a crowded air traffic system exhibits the characteristics of self-organized criticality.

Introduction

WITH the rapid increase of air traffic going into the 21st century, the airspace near major airports will become more and more crowded. There is an urgent need to understand the dynamics of the air traffic system at or near its saturated state.

Current approaches in the analysis of air traffic systems are mainly based on simulation.[1–4] The lack of an analytical approach is because of the difficulty in modeling the complicated interactions among a large number of airplanes. However, there exist some new ideas that have been used successfully in other fields to model complex dynamic systems. One of them is self-organized criticality (SOC).

The concept of SOC was developed by Bak et al.[5] to explain the behavior of complex systems, those containing millions of elements that interact over a short range. Typical examples of such systems are earthquakes, avalanches, stock markets, and ecosystems.[6] Those dynamic systems with many interacting degrees of freedom may organize themselves into a marginally stable critical state. At this SOC state, the system is able to produce a wide range of fluctuations or avalanches. In other words, it can spontaneously generate structures or events of many different sizes. A metaphor for the idea of SOC is a sandpile, which will be described in the next section.

Bak et al. used a simple computer model to simulate the sandpile and concluded that it is SOC. Many other scientists have since conducted experiments on real sandpiles[7–11] and other similar systems such as rice piles.[12] Bak[13] used one chapter in his book to compare all of the experiments. Other authors have applied the SOC concept to various fields. A list of references can be found in Bak's book.[13] However, the SOC concept has not been applied to air traffic problems.

Musha and Higuchi[14] conducted measurements on Japanese highway traffic. The results showed an $1/f$ spectrum in the Fourier-transformed density fluctuations. $1/f$ noise, which has been detected in many physical phenomena such as earthquakes and avalanches, is a classic problem in physics. The original motivation of Bak et al.[5] to develop the SOC model is to explain the widespread occurrence of $1/f$ noise. In recent years, Nagel et al.[15–17] successfully showed that the engineering problems in large, urban highway systems can be explained by SOC.

However, air traffic problems are different from highway traffic problems in many ways. The most significant one is that an airplane cannot stop and go or change its speed as quickly as a car. Therefore, the model used in Nagel's papers cannot be applied to air traffic problems. Another major difference between the two systems is that a car is confined to the highway, while an airplane is free to move in space.

In this paper the concept of SOC is first reviewed using the sandpile model. The dynamic behavior of the air traffic system is discussed next. An air traffic model is examined for the features of SOC. A near-airport simulation is then conducted to demonstrate the model.

Self-Organized Criticality—Sandpile Model

A pile of sand is a deceptively simple model that serves as a paradigm for self-organized criticality. Imagine that a sandpile is built by a device that can place one grain of sand at a time on a flat, horizontal platform. At the beginning, the sand particle just rests where it lands; it is stable. If the sand grains are continuously added to the platform, eventually one particle will land on top of one already there. In this case, it is unstable and the sand grain will fall off to one side. This is a microscopic avalanche.[11]

As sand particles are continuously added to the platform, a sandpile will start to form. When the slope of the pile is small, it is stable and the sand grains will stay where they are sprinkled. As more sand grains are added and the slope of the sandpile increases, it is more likely that the newly added particle will be unstable and start to move after it lands. When one particle moves and hits another particle, it may start a chain reaction, i.e., an avalanche. The avalanche may be small (affects fewer particles) or large (affects more particles). The larger the slope of the sandpile, the higher the possibility of triggering a large avalanche. The slope of the sandpile will decrease after the avalanche. Because the sand grains are continuously added to it, the slope of the sandpile will be gradually reinstalled.

This process will persist until the steady-state angle is reached where, for every particle that is added to the pile, an

Received Aug. 18, 1997; revision received Dec. 8, 1997; accepted for publication Dec. 9, 1997.

*Associate Professor, 3280 Progress Drive. Senior Member AIAA.
†Electronics Engineer, IFSB, 525 Brooks Road.
‡Associate Professor, Department of Physics.

average of one particle will fall off the edge of the platform.[11] In reality, sand grains do not leave the platform one at a time, but rather do so in avalanches. Therefore, the size of the sandpile fluctuates. This phenomenon implies that the sandpile slope has a critical value.[11] If the slope of the pile is smaller than the critical value, the possibility of an avalanche is small, and the addition of sand grains will increase the slope. On the other hand, if the slope of the pile is larger than the critical value, the addition of sand grains will likely trigger large avalanches that will bring the slope back to the critical value. The critical point is self-organized; hence, the name *self-organized criticality*.

There are three major features in an SOC system.[5,6] First, the critical state is robust with respect to any small change in the rules of the system. In the sandpile example, whether the particles are dry sand, wet sand, even snowflakes, the dynamics are similar. Second, the system exhibits fractal structure. In other words, the sandpile has all-length and self-similar avalanches. Third, the system generates $1/f^{\beta}$ noise. If the weight of the sandpile is measured with respect to time, its low-frequency power spectral density displays a power-law behavior over vastly different time scales.

Dynamic Behavior of Air Traffic System

In a highway traffic jam model,[15–17] a car changes its speed reacting to the distance of the car in front of it. When there is a large number of cars in a traffic jam, the complicated interactions among cars can be explained using SOC. The air traffic system is essentially different from the highway traffic system, in that airplanes cannot stop and go or change their speed as fast as cars. They usually follow the preset flying profiles (courses and speeds). The major interaction between two airplanes is collision avoidance. The maneuver involved is mainly the change of direction. Based on this feature, the air traffic system behavior is analyzed in the following text.

When one airplane is flying too close to another airplane, there is a possibility of collision. Both airplanes will receive alarms from their own devices and/or from the air traffic controller. Both pilots have to change their airplane's direction or altitude to avoid collision. To simplify the analysis, we only use direction change so that the problem is reduced to a two-dimensional one. Also, we only consider the sudden encounter of the colliding airplanes. Predicting the colliding possibility from a long distance and changing the course to avoid it is not considered.

If an airplane is moving without any collision possibility, it can maintain its course and is therefore in an undisturbed state, which is analogous to the stable state in the sandpile model. When another airplane appears nearby, the original airplane needs to react to this collision threat, and its state is changed to a disturbed state. As a matter of fact, both involved airplanes are disturbed. If the disturbance moves one of the disturbed airplanes to the vicinity of a third airplane, the third airplane also becomes disturbed. It may start a chain reaction. This process is analogous to the avalanche in the sandpile model. The propagation of disturbance lessens when all airplanes have maneuvered out of collision possibilities and returned to their courses.

How far the disturbance can propagate, or how many airplanes will be disturbed, depends on the local crowdedness of the airspace. If the airspace is overcrowded, the possibility of large disturbances will be high. Once disturbance starts it will persist until some airplanes are driven away. After the crowdedness of the airspace is reduced to a certain level, the disturbance can die down. If the airspace is undercrowded, the disturbance, if any, will be minor and die down quickly. New airplanes can enter the airspace until the number reaches a critical value.

From the preceding analysis, the air traffic system suggests some basic characteristics of self-organization. Next, a generic model will be used to examine the applicability of self-organization criticality to the air traffic system.

Air Traffic Model

We have developed a simple, generic model for this purpose. The airplanes in the model must have the ability to 1) avoid collision; 2) stay in a certain region; and 3) move toward a target, e.g., an airport or a certain direction.

The discrete event model is described as follows. The field is a two-dimensional rectangular grid system. At each step of time, the airplane moves from one grid point to the next one along the grid line (no diagonal motion). When two (or more) airplanes enter the same grid point at the same time, or two entities move over the same grid line at the same time, there is a collision. The possible collision threat positions are shown with x marks in Fig. 1. When at least one other airplane occupies one of those positions the airplane at the center of Fig. 1 is said to be under collision threat(s). The rules that decide how the airplanes move have the following priorities: 1) avoiding collision, 2) staying in the field, and 3) moving toward a preset target or a preset direction. The rules to avoid collision are listed in the Appendix.

The following example is used to illustrate the system behavior of the airspace model described in the previous section. The field and airplanes are shown in Fig. 2. The dotted-line diamond in the figure represents the collision threat positions of airplane No. 11. Because no other airplane is within that diamond, airplane No. 11 is undisturbed. As a matter of fact, all airplanes in the field are undisturbed. All airplanes are flying from the left edge of the field to the right edge of the field. After reaching the right edge, the lifetime of the airplane is over. However, new airplanes appear on the left edge. The field has a stable number of airplanes.

If one airplane, airplane No. 2 (circled by a dotted line), is out of place, as shown in Fig. 3a, airplanes Nos. 4 and 8 are disturbed and have to react to the collision threat. As those three airplanes start to change courses, other airplanes are disturbed, triggering an avalanche. After a period of time the field even-

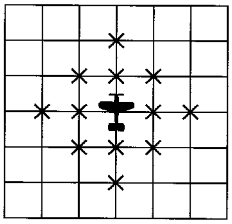

Fig. 1 Possible collision threat positions.

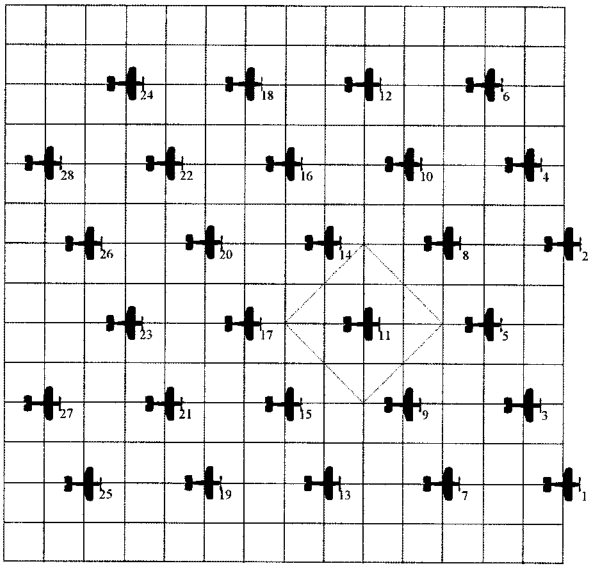

Fig. 2 Example of the undisturbed field. The possible collision threat positions for airplane No. 11 are enclosed by the dotted-line diamond.

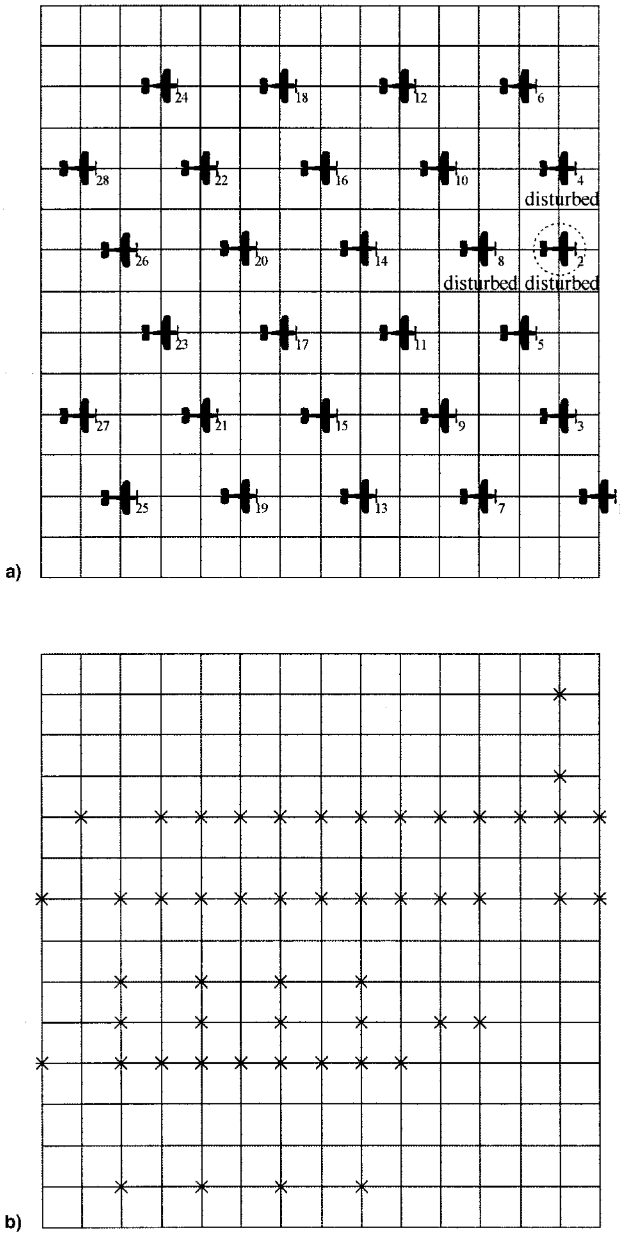

Fig. 3 **a) The field as initially disturbed by airplane No. 2, and b) the positions (marked by x) that are disturbed before the disturbance dies down.**

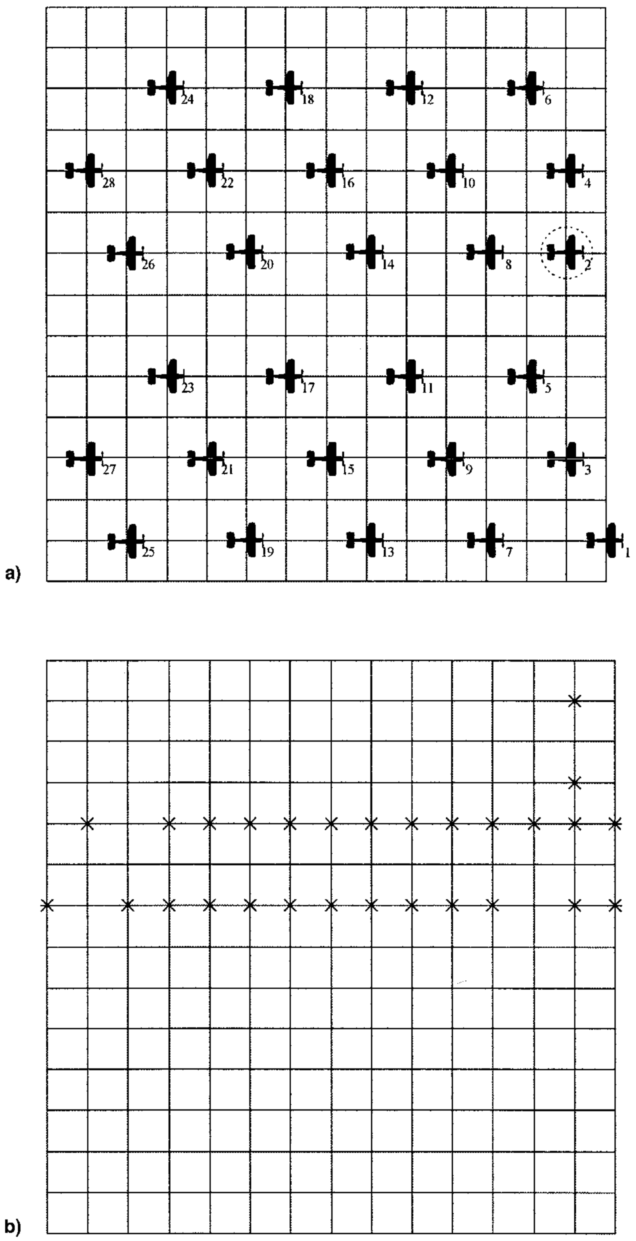

Fig. 4 **a) The field as initially disturbed by airplane No. 2, and b) the positions that are disturbed before the disturbance dies down.**

tually will return to the undisturbed state. Figure 3b shows the positions of the airplanes that are disturbed by the avalanche.

Figure 4 shows another example of field disturbance. In Fig. 4a the airplanes are in a pattern similar to the pattern in Fig. 3a, except that there is a wider gap between the upper and lower groups. The same airplane (No. 2) is out of place. Figure 4b shows the positions that are disturbed before the disturbance dies down. Comparing Fig. 4b with Fig. 3b, it is easy to see that the lower group is completely undisturbed. Apparently, the gap prevents the disturbance from propagating from the upper group to the lower group. Therefore, it is evident that local crowdedness decides the size of the avalanche, which is analogous to the role of the slope in the sandpile model.

If the preceding examples are repeated with different levels of local crowdedness, the number of airplanes that are disturbed will vary, and the final patterns (Figs. 3b and 4b) will be different. Simulation results show that the sizes of avalanches cover a wide range.

Another consequence of the disturbance is that the number of airplanes in the field will fluctuate. The mechanism of the fluctuation is explained as follows. Once an airplane is disturbed and changes its course, the lifetime of the airplane increases; that is, it stays in the field longer because of the detour. The number of airplanes in the field will increase. However, if the disturbance propagates to the left side of the field, new airplanes cannot enter the field because of the collision threats from those disturbed (out-of-place) airplanes. The number of airplanes in the field will decrease. Another possible reason for decreasing numbers, although it did not happen in the examples shown in Figs. 2–4, is that some airplanes may be driven out of the field in the wake of disturbance.

Example: Crowded Airspace near an Airport

Figure 5 shows the field of the airspace near an airport. The field is a 21×21 square. The position of the airport is represented by a circle. After every unit time step (1 s is arbitrarily picked as the time step) one new airplane enters the field from one of the edges. The entering position is randomly picked. All airplanes are flying toward the airport unless they are disturbed. Once an airplane reaches the airport, or is forced out

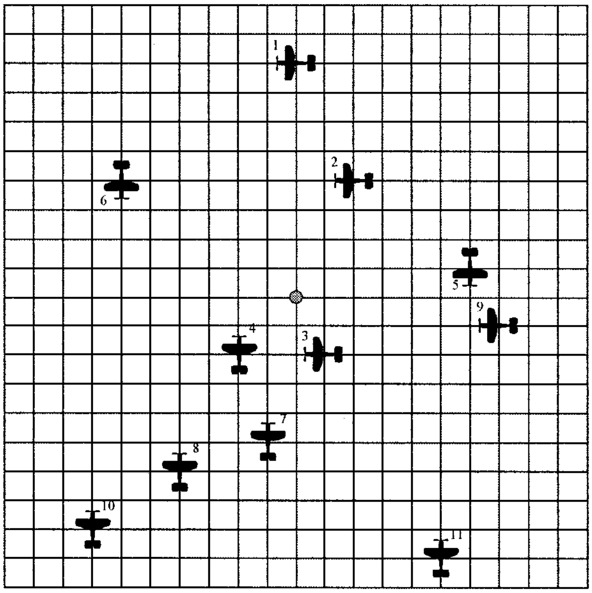

Fig. 5 Field of airspace near an airport.

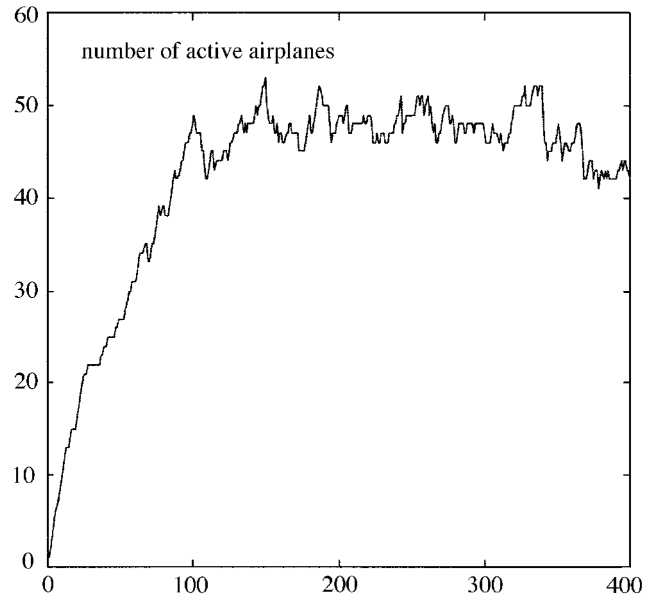

Fig. 6 Number of active airplanes vs time.

of bounds in disturbance, its lifetime is over. The airplanes that are still in the field are called active.

The main difference between this example and the example in Fig. 2 is that, as the airplanes approach the airport, the local crowdedness around the airport increases. After a minor disturbance far away from the airport dies down, the airplanes may again be involved in another disturbance as they fly closer to the airport. As the field is very crowded, there may be many disturbances all over the field. The influence regions of the disturbances overlap each other. It is very difficult to analyze each individual disturbance alone. This is also the major difference between this model and the sandpile model. In the sandpile model, once the avalanche is over, all sand grains will remain at their positions until a new sand grain triggers a new avalanche.

Because of the difficulty in analyzing each individual disturbance, a global approach is used. The total number of active airplanes is recorded at each time step (1 s). This is analogous to the sandpile model, where the weight of the sand on the platform is measured with respect to time. Figure 6 shows the results for the first 400 s. At the beginning, airplanes enter the field at a constant rate and they all stay in the field. The number of active airplanes increases linearly. After a while some airplanes reach the airport, while some are forced out of the field. The number of active airplanes starts to level off. Because the landing rate is lower than the rate of new airplanes entering the field, the field becomes more and more crowded. The scale of disturbance increases and many airplanes are driven off the field. After the crowdedness of the field is reduced, the scale of disturbance is small. The number of airplanes in the field starts to increase again. Therefore, the steady-state value fluctuates. Figure 7 shows the steady-state number of active airplanes for a long period of time. The noise-like fluctuation is evident. Figure 8 shows the power spectral density of the steady-state fluctuation in a log–log diagram. It can be approximated by a straight line with a -1.9 slope. It fits in the model of $f^{-\beta}$ noise. This is another characteristic of SOC.

To examine the robustness of the criticality, the system is changed as follows. The airport in Fig. 5 is closed because of

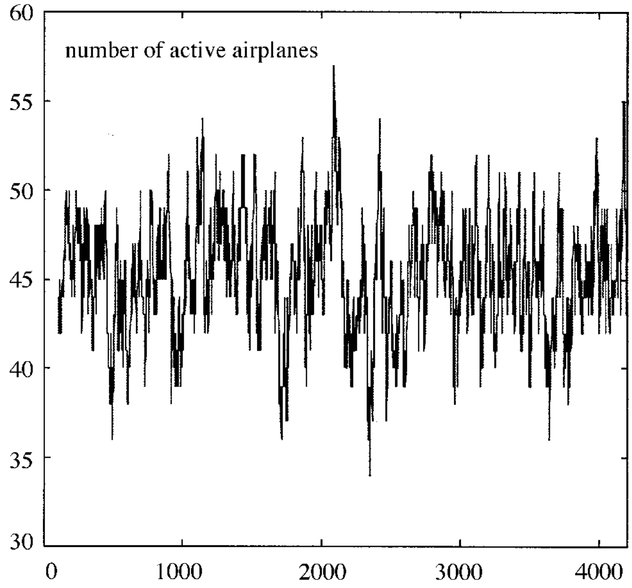

Fig. 7 Steady-state number of active airplanes.

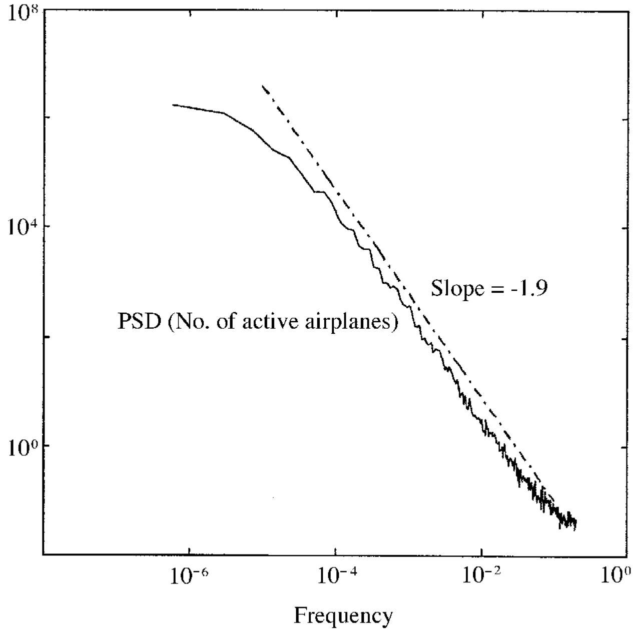

Fig. 8 Power spectral density (128 averages) of steady-state number of active airplanes, airport open.

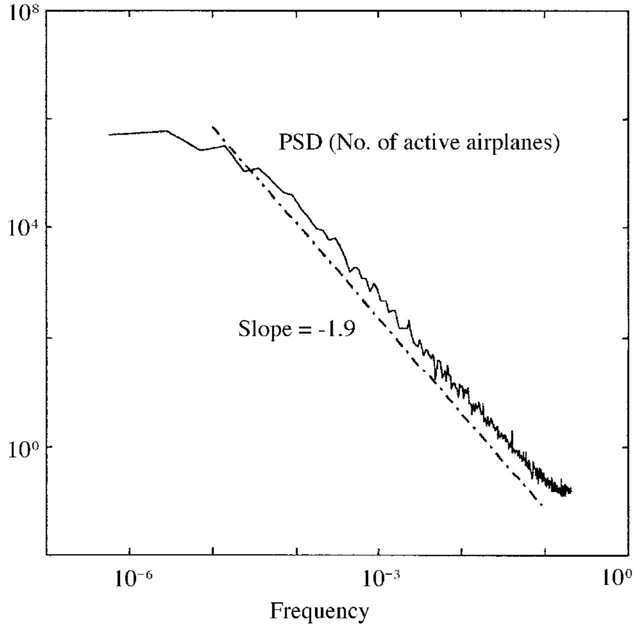

Fig. 9 Power spectral density (128 averages) of steady-state number of active airplanes, airport closed.

bad weather. The airplanes still fly toward the airport but they are not allowed to land. Other conditions remain the same. Figure 9 shows the power spectral density of the steady-state fluctuation in a log–log diagram. The numbers may not be identical, but the fact that it satisfies a power law $f^{-1.9}$ noise profile remains the same.

In the preceding model, the assumption that airplanes enter the field from random directions may not sound very realistic. In real life an air traffic system is usually controlled and scheduled. However, the appearance of an unexpected airplane that disturbs a given airspace is random. The purpose of our model is to study the results of the disturbance in different crowdedness of the airspace. A randomly generated airplane distribution, after a long time, will cover all levels of crowdedness.

Conclusions

This paper has established that the dynamic behavior of a crowded two-dimensional airspace can be explained by a SOC model. The air traffic system looks very different from a sandpile. The sand grains in a sandpile rest at their respective stable positions after a disturbance. In the air traffic system, however, the airplanes keep moving. Despite the difference, they are analogous, as has been shown many times in this paper.

The parameter that decides the critical state in an air traffic system is its local crowdedness. In this paper, the author did not establish a numerical value of the critical local crowdedness. The actual value will depend on the rules of collision avoidance. In the generic model used in this paper, the pattern shown in Fig. 2 is very close to the critical local crowdedness.

The importance of realizing that an air traffic system exhibits the behavior of SOC is that it sets an absolute upper limit for the airspace crowdedness close to an airport. Below this value, the possibility of large disturbance is very low. The actual upper limit used should be lower than this critical value. How much lower will be decided by the tolerance of the disturbance.

In this paper, the authors have only studied two-dimensional airspace. A three-dimensional airspace model can be obtained from the direct extension of the two-dimensional model.

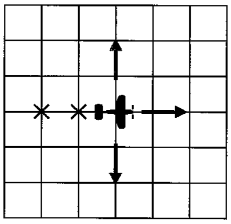

Fig. A1 Disturbance positions and the allowed moves.

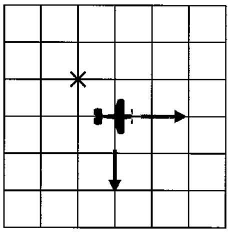

Fig. A2 Disturbance positions and the allowed moves.

Appendix: Rules for Collision Avoidance

When there is an airplane in one of the collision threat positions, the allowed maneuvers are as follows. Figure A1 shows the disturbance coming from one of the x-marked positions. The arrows show the three allowed maneuvers. The rules encourage, but do not require, the airplane to move away from the threat (to the right in the figure). Sometimes, if one of the other two directions takes the airplane closer to the airport, it may be chosen. A weighting factor is added to make the decision.

Figure A2 shows another possible collision threat position and the allowed maneuvers. There is no preference between these two moves. Other factors, such as which move brings the airplane closer to the airport, will decide the maneuver.

Acknowledgment

The research in this paper was partially funded by the U.S. Air Force Office of Scientific Research 1997 Summer Faculty Research Program and Rome Laboratory.

References

[1]Uffelman, K. E., "Flying 800 Planes Simultaneously via Simulation. A Powerful Traffic Simulator for FAA Upgrade of Air-Traffic Control," *Photonics Spectra*, Vol. 25, No. 4, 1991, pp. 108–110.

[2]Blair, E. L., Wieland, F. P., Zukas, A. E., "Distributed Simulation Model of Air Traffic in the National Airspace System," *Proceedings of the 1995 Winter Simulation Conference* (Arlington, VA), IEEE, Piscataway, NJ, 1995, pp. 1203–1206.

[3]Tofukuji, N., "An Airspace Design and Evaluation of Enroute Sector by Air Traffic Control Simulation Experiments," *Electronics and Communications in Japan*, Pt. 1, Vol. 79, No. 8, 1996, pp. 103–113.

[4]Tofukuji, N., "Air Traffic Management Methods: Development and Testing by Real-Time Dynamic Simulation Experiments," *Electronics and Communications in Japan*, Pt. 1, Vol. 80, No. 5, 1997, pp. 35–44.

[5]Bak, P., Tang, C., and Wiesenfeld, K., "Self-Organized Criticality: An Explanation of $1/f$ Noise," *Physical Review Letters*, Vol. 59, No. 4, 1987, pp. 381–384.

[6]Bak, P., and Chen, K., "Self-Organized Criticality," *Scientific American*, Jan. 1991, pp. 46–53.

[7]Bretz, M., Cunningham, J. B., Kurczynski, P. L., and Nori, F., "Imaging of Avalanches in Granular Materials," *Physical Review Letters*, Vol. 69, No. 16, 1992, pp. 2431–2434.

[8]Held, G. A., Solina, D. H., II, Keane, D. T., Haag, W. J., Horn, P. M., and Grinstein, G., "Experimental Study of Critical Mass Fluctuations in an Evolving Sandpile," *Physical Review Letters*, Vol. 65, No. 9, 1990, pp. 1120–1123.

[9]Jaeger, H., Liu, C., and Nagel, S., "Relaxation of the Angle of Repos," *Physical Review Letters*, Vol. 62, No. 1, 1989, pp. 40–43.

[10]Jaeger, H., and Nagel, S., "Physics of Granular State," *Science*, Vol. 255, 1992, pp. 1523–1531.

[11]Nagel, S. R., "Instability in a Sandpile," *Reviews of Modern Physics*, Vol. 61, No. 1, 1992, pp. 321–325.

[12]Frette, V., Christensen, K., Malthe-Sorenssen, A., Feder, J., Jossang, T., Meakin, P., ''Avalanche Dynamics in a Pile of Rice,'' *Nature*, Vol. 379, No. 6560, 1995, pp. 49–52.

[13]Bak, P., *How Nature Works: The Science of Self-Organized Criticality*, Springer–Verlag, New York, 1996.

[14]Musha, T., and Higuchi, H., "The 1/*f* Fluctuation of Traffic Current on an Expressway," *Japanese Journal of Applied Physics*, Vol. 15, No. 7, 1976, pp. 1271–1275.

[15]Nagel, K., and Herrmann, H. J., ''Deterministic Models for Traffic Jams,'' *Physica A*, Vol. 199, No. 2, 1993, pp. 254–269.

[16]Nagel, K., and Paczuski, M., ''Emergent Traffic Jams,'' *Physical Review E*, Vol. 51, No. 4, 1995, pp. 2909–2918.

[17]Nagel, K., Rasmussen, S., and Barrett, C. L., ''Network Traffic as a Self-Organized Critical Phenomena,'' *Self-Organization of Complex Structures: From Individual to Collective Dynamics*, edited by F. Schweitzer, Gordon and Breach, London, 1997, pp. 579–592.